Prediction of Magnetic Flux-Controlled Gate Voltage in Superconducting Field-Effect Transistors

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Abstract—**The unidirectional model of the superconducting field-effect transistor (SFET) is shown to be thermodynamically unsound. A gate voltage which is controlled by the magnetic flux difference in a Josephson weak link is predicted by energy arguments.**

I. Introduction

THE superconducting field-effect transistor (SFET), proposed in [1], has recently been realized in several laboratories [2]–[4]. In the SFET, gate charge controls the critical drain-to-source current by modulating the coupling strength of a Josephson weak link in the region of the transistor channel. Using energy arguments, we show the incompleteness of the unidirectional SFET model, in which gate voltage controls drain current, but not vice versa. We propose a correction which predicts that a change in superconducting current at the drain will result in a change in gate-to-source voltage. This is a static phenomenon which, in Maxwell's equations, requires a constitutive link between the static magnetic vector potential and the electric polarization vector. In Section II we begin by proposing an energy function for the SFET and derive from it the terminal characteristics. In Section III we give a theoretical justification of the correctness of this model. We also show that earlier models, in which the gate charge controls the drain-to-source critical current without a back reaction, violate the laws of thermodynamics.

II. The Model

We describe the SFET in terms of the gate-to-source charge $q_{GS}(t)$ and the drain-to-source flux difference $\phi_{DS}(t)$. By definition, the gate current is $i_{GS}(t) = dq_{GS}(t)/dt$ and the drain-to-source voltage is $v_{DS}(t) = d\phi_{DS}(t)/dt$. We write the energy of the SFET as

$$E(q_{GS}, \phi_{DS}) = \frac{q_{GS}^2}{2C} + \frac{\hbar}{2e} I_0(q_{GS}) \left(1 - \cos \frac{2e}{\hbar} \phi_{DS}\right). \quad (1)$$

The first term is standard and represents the electric energy stored in the gate capacitor. The second term represents the inductive energy stored in the Josephson weak link. It is also standard, except that the inductive energy is a function of the gate charge. This change is necessary to explain the experimental observation of charge-controlled critical current. Because $i_{DS} = \partial E(q_{GS}, \phi_{DS})/\partial \phi_{DS}$, it follows directly that

$$i_{DS}(q_{GS}, \phi_{DS}) = I_0(q_{GS}) \sin \frac{2e}{\hbar} \phi_{DS}. \quad (2)$$

Equation (2) is the Josephson relation except that the critical current is a function of gate charge, as desired. Since $v_{GS} = \partial E(q_{GS}, \phi_{DS})/\partial q_{GS}$, we also find that

$$v_{GS}(q_{GS}, \phi_{DS}) = \frac{q_{GS}}{C} + \frac{\hbar}{2e} \omega_T(q_{GS}) \left(1 - \cos \frac{2e}{\hbar} \phi_{DS}\right) \quad (3)$$

where ω_T is the transition frequency, defined as

$$\omega_T(q_{GS}) \equiv \left[\frac{dI_0(q)}{dq}\right]_{q_{GS}}. \quad (4)$$

For $\omega_T = 0$, we return to the usual relations, in which the gate voltage is just proportional to the gate charge; however, when the critical current depends on the gate charge, we discover that the gate voltage must depend on the drain-to-source flux ϕ_{DS}. For conventional transistors ω_T is the transconductance g_m divided by the input capacitance C, and is a common speed indicating parameter.

An order-of-magnitude estimate of the voltage pre-factor $V_b = \hbar\omega_T/2e$ can be obtained by assuming a simple model in which the product of the critical current and normal resistance is constant. Assume $V_0 = I_0(q_{GS})R_{DSn}(q_{GS})$, where V_0 is a voltage generally less than the superconducting energy gap and R_{DSn} is the MOSFET output resistance, in the linear regime, given by $R_{DSn} = L^2/(\mu q_{GS})$, where L is the transistor length and μ is the carrier mobility. This yields $\omega_T = \mu V_0/L^2$. For the values $L = 100$ nm, $V_0 = 1$ mV, and $\mu = 10000$ cm^2/V·s, we obtain $\omega_T = 10^{11}$ s^{-1} and $V_b = 33$ μV. For a SFET with a silicon dioxide gate insulator of thickness 10 nm and width 100 μm, we find $C = 35$ fF and the charge $V_bC = 1.1 \times 10^{-18}$ C, equivalent to a surface charge density of only 7×10^7 cm^{-2}. From an experimental standpoint, the voltage V_b is easily observable; however, a fairly large array of devices, with stable surface traps, will be needed to achieve a measurable dc charge. Significant high-frequency currents are more easily attained but more difficult to distinguish from crosstalk. It is expected that V_b will be important in integrated circuit configurations and, indeed, the use of on-chip sense circuitry may be the best approach to measuring V_b.

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III. The Theory

There are many ways to frame energy arguments about physical devices. Our approach is to use the formalisms of circuit theory. The key point we must address is why the characteristics of an SFET are controlled by an energy function such as (1). Given (1), the existence of the gate voltage effect follows immediately, but first we need to show why such a function must exist. After all, this argument would be completely invalid if applied to a MOSFET. The argument we use is actually quite simple—only if we can describe the SFET in terms of an energy function is the device passive. Otherwise, it can supply infinite net energy. This result is easily proved within the framework of a circuit theory formulation.

From a circuit theoretic standpoint, the SFET can be modeled as a generalized capacitive/inductive n-port [5], [6]. In this algebraic model, the SFET has no internal dynamics, just as is the case with a multiport nonlinear inductor or capacitor. We define a state column vector $\boldsymbol{x} = (q_{GS}, \phi_{DS})^T$, where q_{GS} is the gate-to-source charge and ϕ_{DS} is the drain-to-source flux difference. The motion of this vector is controlled by voltages and currents applied to the ports of the SFET. We have

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{u} = (i_{GS}, v_{DS})^T. \tag{5}$$

Note the circuit-oriented definition of flux. In this paper (magnetic) flux is defined as the time integral of voltage; it is equal to $\hbar/2e$ times the superconducting phase difference across the Josephson junction. The constitutive law for the device is

$$\boldsymbol{y} = (v_{GS}, i_{DS})^T = \boldsymbol{f}(\boldsymbol{x}), \tag{6}$$

where $\boldsymbol{f}(\cdot)$ is a continuous vector field on the reals and $\boldsymbol{u}$ and $\boldsymbol{y}$ form a hybrid pair, that is, a pair of vectors whose inner product is power. Assuming associated reference directions, the power entering the device is $\boldsymbol{u}^T\boldsymbol{y}$. Wyatt *et al.* [5] show that this device is not active iff $\boldsymbol{f}(\cdot)$ can be written

$$\boldsymbol{f}(\boldsymbol{x}) = \nabla E(\boldsymbol{x}) \tag{7}$$

where $E(\cdot)$ is a differentiable scalar function bounded from below. By "active" we mean in the sense that a battery is active rather than in the sense that a transistor can be biased in a regime where it is incrementally active. It was also shown that if a generalized capacitive/inductive n-port is not active then it is lossless [6] and the Jacobian of $\boldsymbol{f}$ is symmetric when evaluated at every constant operating point $\boldsymbol{x}_0$. When $E(\cdot)$ exists, the energy absorbed by the device over the time interval $[t_1, t_2]$ is $E(\boldsymbol{x}(t_2)) - E(\boldsymbol{x}(t_1))$, which is a function only of the state. Note that between any two times at which $\boldsymbol{x}(t_2) = \boldsymbol{x}(t_1)$, the net energy absorbed by the device is zero. For either a generalized capacitive or inductive n-port, symmetry of the Jacobian implies reciprocity, but reciprocity is not implied in the present hybrid device. This fact is particularly important in light of the difficulty of designing nonreciprocal Josephson devices [7].

If the SFET model is assumed unidirectional (the gate controls the drain but not vice versa, and the Jacobian is asymmetric), then an energy function does not exist. Such a model predicts that the SFET is capable of supplying infinite energy. This energy can be extracted in a small-signal manner either by biasing the device at a drain current between 0 and I_0 and traversing an appropriately directed closed path in the phase plane near $\boldsymbol{x}_0$, or by biasing the device at a nonzero drain-to-source voltage and using a large-signal gate pump, such as $I_0(q_{GS}) = 2 - \sin 2e\phi_{DS}(t)/\hbar$. Adding a series gate resistor and a parallel drain/source resistor (e.g., the transistor's normal output resistance) to the model makes the thermodynamic failure of the unidirectional model less violent (one can no longer extract infinite power), but it is still always possible to extract net energy by using low drain voltages and slowly varying pump waveforms. From this we conclude that the SFET is not a unidirectional device and must be described by an energy function.

With $E(\cdot)$ given by (1), (2)–(4) follow from (5)–(7). Note that any form of $E(\cdot)$ which results in the prediction of a charge-controlled critical current, e.g., (2), leads to a flux-controlled gate voltage. Since charge-controlled critical currents are experimentally observed [2]–[4], the flux-controlled gate voltage effect must exist.

Physically, the gate voltage effect is a direct consequence of energy conservation. Normal mobile charges in the channel region influence both the capacitive and inductive components of the energy. In a lossless system, and the SFET can always be driven sufficiently slowly that resistive losses are made arbitrarily small, the existence of charge-controlled inductive energy must lead to symmetric coupling between flux and charge. If one were to perform the experiment of moving a charge δ from the channel to the gate by applying a gate voltage, this would require changing the capacitive energy of the SFET by $q_{GS}\delta/C$ and the inductive energy by $\delta(\hbar\omega_T/2e)(1 - \cos 2e\phi_{DS}/\hbar)$. The flux-dependent change in inductive energy induced by the change in channel charge is exactly the term that appears in the gate voltage expression. The origin of the energy function was chosen so that the change in inductive energy is zero if $\phi_{DS} = 0$. An electromechanical analogy may help to clarify the physics: imagine a two-terminal vacuum Josephson tunneling junction with an electrostatic motor/generator to control the separation of the superconducting plates. We neglect friction. In equilibrium, where $\phi_{DS} = i_{DS} = 0$, it requires no energy to move the plates, but for $\phi_{DS} \neq 0$, moving the plates requires changing the inductive energy of the system and this requires applying a voltage to the motor. If, instead, one changes the flux ϕ_{DS}, this also changes the inductive energy of the system, exerts a force on the plates, and causes a voltage to appear on the motor. This is the physics behind the voltage effect discussed above.

In conclusion, we have shown that the unidirectional SFET model is thermodynamically unsound. For a passive SFET model to be consistent with recent experimental observations of a charge-controlled critical current, a back-reaction from the dc drain-to-source flux (phase difference) to the dc gate voltage is required. As this effect is important in large devices and occurs at $v_{DS} = 0$, it does not appear to be directly related to charge space energy bands [8]–[11] or quasi-particle interference [12]–[14].

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